Testing of various parametrizations of effective nucleon-nucleon interaction in elastic and inelastic scattering of polarized protons from ^{12}C

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The parametrization schemes of the effective nucleon-nucleon interaction proposed by Geramb and by Nakayama and Love are compared in their ability to reproduce the inelastic scattering observables of polarized protons from ^{12}C with the excitation of the 1^+ level with $T = 1$. The off-shell behavior of the effective t matrix is shown to be important. Density dependence of the effective interaction is also analyzed.

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I. INTRODUCTION

The inelastic scattering of protons from nuclei is a useful tool of investigating nuclear properties. It is necessary to know the effective NN interaction between the projectile proton and nucleons in the nucleus to extract information about the properties of its excited states. The problem of the modification of the free NN interaction in the nuclear medium has been studied in many papers (see e.g. [1, 2, 3, 4, 5]). The effective NN interaction can be used in local density approximation for the description of elastic [6, 9] or inelastic [9, 10, 11] scattering.

For proton energies in the range from 100 to 400 MeV, the main modification of the free NN scattering reaction t matrix in the nuclear medium is due to the Pauli exclusion principle for scattering in occupied states and due to the influence of the mean nuclear field inside the nuclear matter. It is the g-matrix function that is used to quantitatively describe this influence. The g matrix is a solution of the Bethe-Goldstone equation [12] - the equivalent of the Lippman-Swinger equation for the t matrix in the nuclear medium. There are two approaches to the solution of this equation which are different: one is based on coordinate space [2, 12] and the other one on impulse space [3, 13, 14]. The parametrization by von Geramb [2] was especially successful. It provides the so- called Paris-Hamburg (PH) effective interaction which describes elastic and inelastic scattering uniformly well for nuclei of different atomic weight.

Later a new parametrization of the effective interaction was proposed by Nakayama and Love (NL) [3]. In this work the authors achieved some success in the description of high-spin stretched states in ^{28}Si. In work [4] it was claimed that the NL construction better reproduces the low density behavior of the tensor component of the effective interaction as compared with the free NN interaction and it is important for the success in the case of stretched states. In the present paper we hold the converse opinion - that the similarity between the NL interaction and the free NN interaction is a disadvantage of the NL construction. We compare the predictions of inelastic scattering observables based on these two parametrizations of the effective interaction with the experiment and show that the Geramb interaction reproduces them better. The reason for the success of the Geramb forces is also analyzed.

II. ELASTIC SCATTERING

The optical potential of the nucleon in nuclear matter can be calculated in the first order microscopically [6]. The Geramb and NL g matrices lead to optical potentials considerably different in their radial form. They are sensitive only to the isoscalar components of the g matrix. The elastic observables, differential cross sections ($d\sigma/d\Omega$), analyzing power (A_y), and the spin-rotation parameter (Q) calculated using these potentials are represented in Fig.1.

As is seen from the comparison with the experimental data [7], the Geramb g matrix better reproduces the experiment. The reason for the preference of the Geramb forces over the NL ones in the isoscalar channel was explained in work [4]. As different optical potentials influence the calculated inelastic observables, further we use the phenomenological optical potential [8] for the calculation of distorted waves for the inelastic channel. This potential satisfactorily reproduces the experimental observables (see Fig.1) of elastic scattering.

III. CONSTRUCTION OF THE EFFECTIVE INTERACTION

Von Geramb, Nakayama and Love used a Bethe-Goldstone type equation for the infinite nuclear matter to obtain the g-matrix interaction of a free nucleon with a nucleon in the nuclear medium. The nuclear matter is described
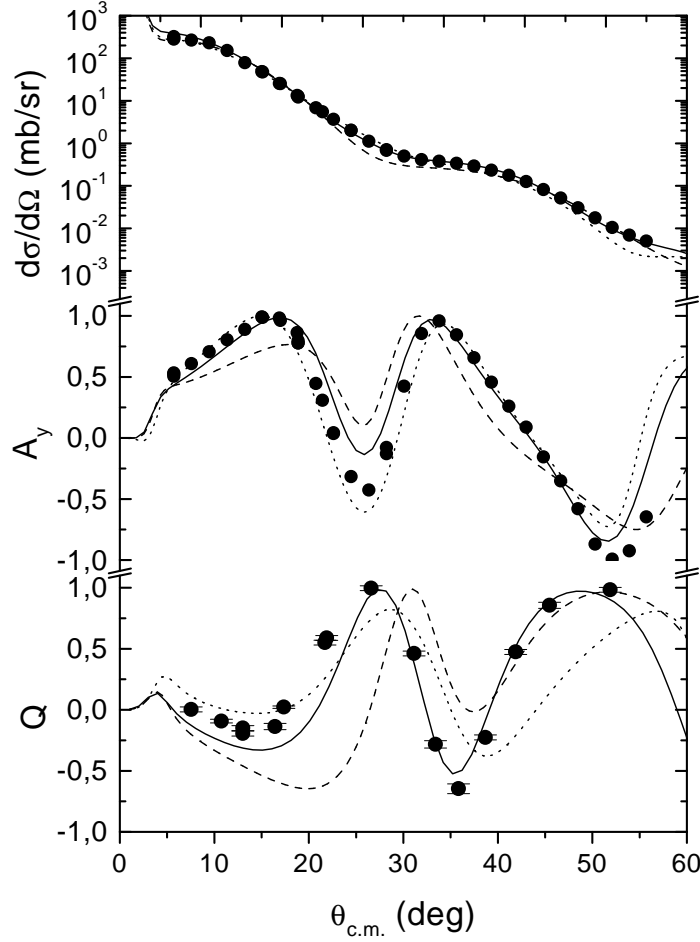
FIG. 1: Comparison of the calculated elastic scattering observables $d\sigma/d\Omega$, A_y and Q with the experiment. The experimental data are taken from [7, 8]. Calculations (DWBA-91): the solid lines show the microscopical calculations using the Geramb interaction, dashed lines show the same, using the NL interaction, and the dotted lines indicate calculations, using the phenomenological WS potential.

by non-interacting Fermi gas with the Fermi impulse k_F. The difference was that Nakayama and Love solved this equation in an impulse representation while von Geramb worked in coordinate space.

Lets assume $\mathbf{k}_0$ and $\mathbf{k}$ to be relatively initial and final relative momenta of two nucleons and $\mathbf{K}$ as their center-of-mass momentum. The Bethe-Goldstone equation then will have the following form [15]:

$$G(\mathbf{K}|\mathbf{k}, \mathbf{k}_0) = V(\mathbf{k}, \mathbf{k}_0) + \int d^3k \frac{V(\mathbf{k}, \mathbf{k}')Q(\mathbf{K}, \mathbf{k}')G(\mathbf{K}|\mathbf{k}', \mathbf{k}_0)}{E(\mathbf{K}, \mathbf{k}_0) - E(\mathbf{K}, \mathbf{k}') + i\epsilon}, \tag{1}$$

where $Q(\mathbf{K}, \mathbf{k}')$ satisfies the Pauli principle condition: $Q(\mathbf{K}, \mathbf{k}') = 1$ for $|\mathbf{K} + \mathbf{k}'| > k_F$ and $Q(\mathbf{K}, \mathbf{k}') = 0$ for $|\mathbf{K} \pm \mathbf{k}'| < k_F$. The function $E(\mathbf{K}, \mathbf{k})$ in the denominator denotes the sum of single-particle energies of the two nucleons. Usually the operator Q in the equation (1) is replaced by its angle averaged value [15]

$$\overline{Q}(K, k') = \frac{1}{4\pi} \int d\hat{k}' Q(\mathbf{K}, \mathbf{k}').$$

The energy denominator in the equation (1) is also substituted by its angle averaged form though the prescription of this averaging is different in the Geramb [2] and NL [3] papers. This difference was discussed in [4] and we do

not analyze it here because it is absent for the case of $k_F = 0$. As we show later, the g matrices obtained by von Geramb and Nakayama and Love are also different in this particular case and it is the main reason for the differences in predicted inelastic observables we consider based on these two interactions.

With the angle averaged Pauli operator Q and the denominator the solution of the Bethe-Goldstone equation does not depend any more on the direction of the vector $\mathbf{K}$. The remaining angular dependent part of the function G can be expanded using partial-wave decomposition

$$G(K|\mathbf{k}, \mathbf{k}_0) = \frac{2}{\pi} \sum_{\substack{JST \\ LL',M}} i^{L-L'} G_{L'L}^{JST}(K|k, k_0) T_{LSJ,M}(\hat{k}_0)^* T_{L'SJ,M}(\hat{k}) P_T, \tag{2}$$

where the spin-angle function T_{LSJ} is an eigenstate of two nucleons with the total angular momentum J, total spin S and the total orbital angular momentum L. The operator P_T is a projector on the state of two nucleons with the total isospin T. The substitution of the decomposition (2) by (1) leads to a coupled-channel integral equation for the matrix elements $G_{L'L}^{JST}(K|k, k_0)$. It can be numerically solved for each channel with the quantum numbers JST. This method was developed by Nakayama and Love.

Von Geramb solved a related Bethe-Golstone equation for the correlated wave function (WF) of two nucleons $\psi(\mathbf{r}, K, \mathbf{k}_0)$. This equation has the following form [9]:

$$\psi = \phi + G_Q^{(+)} V \psi, \tag{3}$$

where V is the free NN interaction, ϕ is the plane wave function of the relative NN motion and $G_Q^{(+)}$ is the Green function of two nucleons in the nuclear matter with outgoing wave boundary conditions

$$G_Q^{(+)} = \sum_{|\mathbf{k_1}|,|\mathbf{k_2}|>k_F} \frac{Q(\mathbf{k}_1, \mathbf{k}_2)}{E - e_1 - e_2 + i\epsilon}.$$

Here Q is the Pauli projecting function for two nucleons in intermediate states with the wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$, e_1, e_2 are single particle energies of nucleons in the nuclear medium.

The g matrix and ψ are connected with the following expression:

$$< \phi|G|\phi > = < \phi|V|\psi >.$$

This equation allows to interpret the g matrix as an effective NN interaction in the nuclear matter. The numerical solution of equation (3) is performed with the expansion of the WF ψ over the multipoles by

$$\psi(\mathbf{r}, K, \mathbf{k}_0) = \sum_{\substack{JLL' \\ ST}} (4\pi(2L+1))^{\frac{1}{2}} i^L u_{L'L}^{JST}(r, k_0, K) \tag{4}$$

$$\times T_{L'SJ,M}(\hat{r}, \hat{k}_0) < L0SM|JM > P_S P_T$$

and with the use of well-known techniques for the solution of wave equations.

In both considered cases the desired local complex energy- and density-dependent effective interaction has the following operator form:

$$V(r) = \sum_{ST} V_{ST}^C(r) P^S P^T + \sum_T V_T^{LS}(r) \mathbf{L} \cdot \mathbf{S} + \sum_T V_T^T(r) S_{12}. \tag{5}$$

Here L is a total orbital angular momentum operator and S_{12} is a usual tensor spin operator. The antisimmetrized matrix element of this force between two nucleon states with the relative wave vectors $\mathbf{k}_0$ and $\mathbf{k}$ in the initial and final channels respectively has the following form [4, 17]:

$$\tilde{t}(\mathbf{k}, \mathbf{k}_0) = \sum_{ST} \tilde{t}_{ST}^C P_S P_T + i \sum_T \tilde{t}_T^{LS} (\sigma_1 + \sigma_2) \cdot \hat{\mathbf{n}} P_T \tag{6}$$

$$- \sum_T [\tilde{V}_T^T(q) S_{12}(\hat{\mathbf{q}}) - (-)^{1+T} \tilde{V}_T^T(Q) S_{12}(\hat{\mathbf{Q}})] P_T,$$

where $\mathbf{q} = \mathbf{k}_0 - \mathbf{k}$, $\mathbf{Q} = \mathbf{k}_0 + \mathbf{k}$, $\hat{\mathbf{n}} = \hat{\mathbf{q}} \otimes \hat{\mathbf{Q}}$, and where

$$\tilde{t}_{ST}^C(q) = \tilde{V}_{ST}^C(q) - (-)^{S+T}\tilde{V}_{ST}^C(Q), \tag{7}$$

$$\tilde{t}_T^{LS}(q) = \frac{1}{4}qQ[\tilde{V}_T^{LS}(q) + (-)^{1+T}\tilde{V}_T^{LS}(Q)].$$

The functions $V^{C,LS,T}(q)$ are the Fourier transforms of the corresponding radial functions in (5) [4, 17].

When Nakayama and Love were constructing various components of radial dependent functions in (5) from the matrix elements $G_{L'L}^{JST}(K|k,k_0)$, they used these matrix elements only on-shell ($k = k_0$). Using some simplifications, the wave vectors K and k can be evaluated via the wave vector of the projectile k_1 and the Fermi momentum k_F. If θ is the scattering angle in the c.m. frame, then the momentum transfer q is equal to

$$q = 2k_0 sin\frac{\theta}{2}.$$

The spin-dependent antisimmetrized part of the effective interaction in the central channel for example can be obtained through the following equation:

$$G_1^C(q, k_1, k_F) = \frac{1}{2\pi^2}\sum_{LJ}(2J+1)G_{LL}^{J1T}(K|k,k)P_L(cos\theta)\frac{1}{3}P_{S=1}.$$

The summation in this equation over L is restricted by the Pauli principle. This interaction can be further expanded into isoscalar and isovector parts. Nakayama and Love identified then the isovector component with the following combination of NN t-matrix elements of equation (6):

$$G_1^C(q, k_1, k_F) = \eta(\tilde{t}_{00}^C(q, k_1) - \tilde{t}_{01}^C(q, k_1) - \tilde{t}_{10}^C(q, k_1) + \tilde{t}_{11}^C(q, k_1))/16, \tag{8}$$

where η is a Jacobian of transformation from the NN center of mass to the NA center of mass [17]. In the same manner Nakayama and Love expressed other components of effective interaction (6) via a corresponding combination of g matrix elements. The desired effective interaction was obtained in a fitting procedure for different energies of the projectile and Fermi momentum k_F.

Von Geramb, however, used a different fitting procedure. Following [1] he identified the g matrix effective interaction with the averaged two nucleons free NN interaction by using plane incoming and correlated outgoing wave functions. This procedure reproduces in general radial correlations of two-body WF in the nuclear matter carried by ψ:

$$V_{ST}^C(r) = \frac{\sum_J \hat{J}(LJST|t|LJST)}{\hat{S}\hat{L}}, \tag{9}$$

where

$$(L'JST|t|LJST)$$
$$= \frac{\sum_{L"}\int_{|\mathbf{p}|\leq k_f}d^3p j_L(k_0 r)V_{L"L}^{JST}(r)u_{L'L"}^{JST}(r, k_F, k, K)}{\int_{|\mathbf{p}|\leq k_f}d^3p j_{L'}(k_0 r)j_L(k_0 r)}. \tag{10}$$

The central component of the effective interaction obtained according to (9) depends on the orbital momentum L. Additional averaging was performed with plane wave states. The effective interaction obtained in this manner was approximated in terms of Yukawa form factors.

IV. COMPARISON OF EFFECTIVE ISOVECTOR INTERACTIONS

To compare the Geramb and NL interactions, we reproduce in Fig.2 the real and imaginary parts of the central isovector spin-dependent, isovector spin-orbit and direct isovector tensor components of the t matrix (Eq. (6)) for $k_F = 0$. The spin-orbital component of the force $\tilde{\tau}^{LS}$ is expressed through $\tilde{t}^{LS}$ (Eq. (7)) in the following equation:
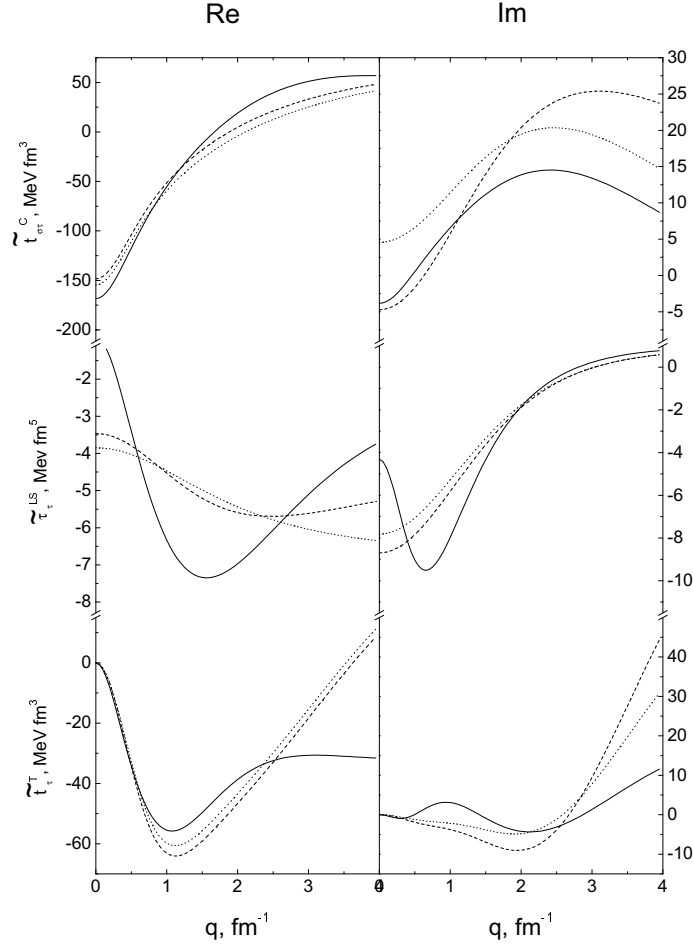
$$\tilde{\tau}^{LS}(q) = -\frac{1}{qQ}\tilde{t}^{LS}(q)$$

FIG. 2: The q dependence of different components of the effective t-matrix interaction (real - the left-hand side and imaginary - the right-hand side respectively). The solid lines show the Geramb interaction, dashed lines indicate the NL, and the dotted lines show the FL free interaction.

Fig. 2 also shows the t matrix obtained from the Franey and Love (FL) parametrization of the free NN interaction [16].

As is seen from the comparison of the real parts of different components of the NL and FL interactions, they are similar, while the Geramb interaction is considerably different from both. The imaginary part of the NL interaction is also more similar to the FL one and both of them are notably different from the Geramb interaction. As in work [4], we believe that this is due to an approximate taking into account of the off- shell behavior in the Geramb scheme of the approximation of the g matrix by the local Yukawa form potential and the lack of it in the NL scheme.

V.   DESCRIPTION OF INELASTIC SCATTERING OF POLARIZED PROTONS WITH THE EXCITATION OF THE $1^+, T = 1$ LEVEL IN ^{12}C USING VARIOUS EFFECTIVE INTERACTIONS

The excitation of the level $1^+, T = 1$ with $E^* = 15.11$ MeV in ^{12}C is produced with those isovector spin-dependent components of the force (5) that we have investigated in the above section. We used the Cohen-Kurath wave function for the description of the excited state [18]. It satisfactorily reproduces the $B(M1)$ value of this level as well as the momentum dependence of the transverse formfactor of (e, e') scattering [6]. The calculations of the inelastic scattering observables were performed in the framework of Raynal's program DWBA-91 [19]. It takes into account
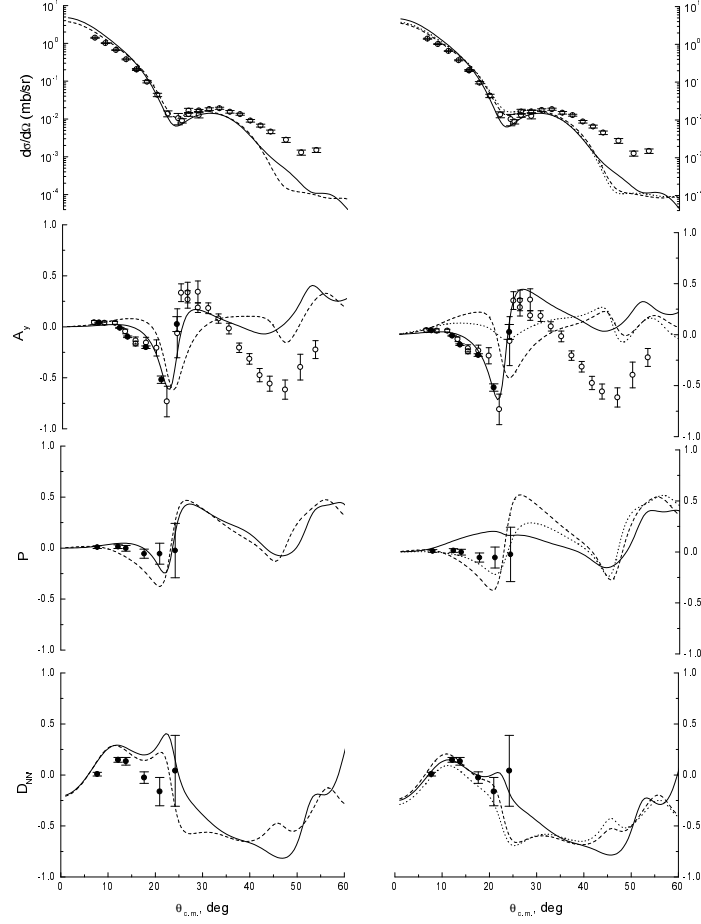
FIG. 3: Comparison of the calculated differential cross-section $(d\sigma/d\Omega)$, analyzing power (A_y), polarization (P) and polarization transfer coefficient $D_{NN'}$ for the level 1^+ in ^{12}C with the experiment $(E_p \simeq 200$ MeV). The open circles indicate the data taken from [7], the solid curves show the results from [20]. The left-hand side represents density dependent calculations, the right-hand side shows those without density dependence. The solid lines show the Geramb interaction, dashed lines indicate the NL, and the dotted lines show the FL interaction.

the antisimmetrization of the WF of the incident proton and active nucleons in the nuclear. We tested the Geramb and NL forces as effective interactions. Their comparison with the experiment is presented in Fig.3. The experimental differential cross-section $(d\sigma/d\Omega)$ and part of the analyzing power (A_y) data were obtained in work [7]. The energy of scattered protons in this experiment was 200 MeV. The remaining data of the analyzing power , polarization (P) and the polarization transfer coefficient $D_{NN'}$ were taken at 198 MeV from work [20]. $D_{NN'}$ is only one normal-component polarization transfer observable.

Two cases were considered with the use of density dependence and without it. They are presented on the left- and right-hand side of Fig.3 respectively. For the case without density dependence, we have also presented calculations with the use of the FL parametrization of the free t matrix of the NN interaction. As can be seen from the comparison of the density dependent calculations with the experiment, the NL interaction describes the cross-section almost as well as the Geramb interaction. The deterioration in the description of the experiment with the increase of the scattering angle is also seen in inelastic electron scattering, where the WF of Cohen and Kurath do not describe properly the second maximum of the transverse form factor [6]. The behavior of the cross-section at small angles is following the absolute square of the $t^C_{\sigma\tau}$ component which is very similar in both cases. The behavior of the A_y and P is different for the Geramb and NL interactions. The Geramb interaction better reproduces the experimental

data. The shift of the NL analyzing power towards large angles (see also [3]) is the consequence of different behavior of t^{LS} and t^T versus q in both parametrizations. This difference is further amplified when the density independent interactions are used (the right-hand side of Fig.3). Nearly the same situation is seen for polarization. In both cases the use of the FL interaction in our calculations leads to a similar behavior of the analyzing power and polarization as with the NL interaction.

The calculated polarization transfer coefficient is similar for both the Geramb and NL interactions. It overestimates the experimental data in both cases. Though this overestimation is absent for density independent forces, this fact just reflects some uncertainty. On the whole the DWBA calculations with the Geramb interaction better reproduce the experimental data than with the NL and FL one.

VI. CONCLUSION

We examine the Geramb and NL effective interactions in their ability to reproduce the excitation of the isovector anomaly parity state 1^+ in ^{12}C with polarized protons. This excitation is driven with the isovector spin-dependent components of the effective NN interaction. As is seen from the construction of the effective interaction by von Geramb, it takes into account its off-shell behavior approximately. On the other hand, the construction of the NL interaction completely ignores the off-shell behavior of the g matrix - the solution of the Bethe-Golstone equation. We think that it is this advantage of the Geramb (PH) interaction that allows to reproduce satisfactorily the polarization observables of inelastic scattering. The elastic sattering calculations using the PH model are especially successful in the description of the elastic observables, including the triple-scattering parameter Q (Fig. 1). The good description of the latter is rather remarkable for nonrelativistic theories.

Acknowledgements

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